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DESIGN DATA FOR MULTIPOST-STIFFENED
WINGS IN BENDING

By Roger A. Anderson, Aldie E. Johnson, Jr.,
and Thomas W. Wilder, III

Langley Aeronautical Laboratory
Langley Field, Va.



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In equation (4) on page 6, the first two terms in the denominators of the expressions following the two summation signs should be placed within brackets and squared so that the equation reads as follows:

$$\frac{1}{\left(\frac{\lambda}{b}\right)^4 T} + \frac{1}{\gamma_C - \left(\frac{\lambda}{b}\right)^2 \delta_C k_C} + \frac{1}{\sum_{r=-\infty}^{\infty} \frac{1}{\left[1 + \left(2r + \frac{p}{n}\right)^2 \left(\frac{\lambda}{b}\right)^2\right]^2 - \left(\frac{\lambda}{b}\right)^2 k_C}} +$$

$$\frac{D_C}{D_T} \frac{1}{\gamma_T + \left(\frac{\lambda}{b}\right)^2 \delta_T k_T} + \frac{1}{\sum_{r=-\infty}^{\infty} \frac{1}{\left[1 + \left(2r + \frac{p}{n}\right)^2 \left(\frac{\lambda}{b}\right)^2\right]^2 + \left(\frac{\lambda}{b}\right)^2 k_T}} = 0 \quad (4)$$



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SUMMARY

The results of a computational program are presented which give numerical values of the stiffnesses required of the various components of a multipost-stiffened wing to achieve desired buckling-stress values under bending loads. Two arrangements of the posts are considered, upright posts and posts used as diagonals of a Warren truss. This work extends and summarizes the calculations presented in NACA RM L52K10a.

INTRODUCTION

Upright members known as posts have been used to advantage for providing local stabilization between the tension and compression surfaces of wing structures in a number of production aircraft. In these applications the posts usually are employed in locations such as fuel-cell bays where stress analyses or strength tests indicate additional stiffness to be necessary. More recently, the use of a systematic arrangement of posts between the wing surfaces has been proposed (ref. 1) as a partial solution to some of the fabrication problems associated with thin wings. This arrangement has been called the multipost-stiffened wing and may be described as essentially a multiweb structure in which alternate full-depth webs have been replaced by lines of small stringers connected by posts at appropriate intervals. The present paper is concerned with an analysis of the interaction of the various stiffnesses which influence the buckling behavior of such a structure.

A stability criterion has been derived in reference 2 which takes into account the essential stiffnesses that must be considered in an analysis of the multipost-stiffened structure under a bending moment. The criterion was used in reference 3 to make a limited set of calculations to determine the stiffnesses required of combinations of stringers and posts to achieve desired buckling-stress coefficients for the structure. This work has been extended and is summarized in the present paper. In addition to providing stiffness data for a structure stiffened

by vertical posts, the paper provides information on the required stiffnesses when the posts are inclined to form a Warren truss and also when transverse stiffeners or formers are used to provide the equivalent of post stiffness at certain locations.

SYMBOLS

σ_C	longitudinal stress in compression cover of beam at buckling
σ_T	longitudinal stress in tension cover of beam at buckling
k_C	compressive-buckling-stress coefficient, $\frac{\sigma_C t_C b^2}{\pi^2 D_C}$
k_T	tensile-stress coefficient, $\frac{\sigma_T t_T b^2}{\pi^2 D_T}$
A	cross-sectional area of stringer
b	width of cover bay between longitudinal lines of support
t_C	thickness of compression cover skin
t_T	thickness of tension cover skin
δ_C	area ratio of stringer to cover skin on compression side of beam, A/bt_C
δ_T	area ratio of stringer to cover skin on tension side of beam, A/bt_T
l	length of cover bay between post supports
β	post-spacing ratio, l/b
λ	length of buckle (distance between transverse nodes)
E	Young's modulus of elasticity
μ	Poisson's ratio
EI	flexural stiffness of stringer

D_C	flexural stiffness of compression cover, $\frac{Et_C^3}{12(1 - \mu^2)}$
D_T	flexural stiffness of tension cover, $\frac{Et_T^3}{12(1 - \mu^2)}$
γ_C	flexural-stiffness ratio of stringer to cover skin on compression side of beam, $\frac{EI}{bD_C}$
γ_T	flexural-stiffness ratio of stringer to cover skin on tension side of beam, $\frac{EI}{bD_T}$
F	effective spring stiffness of post-stringer combination, force per unit extension
T	effective post-stiffness parameter, $\frac{F}{l} \frac{b^3}{\pi^4 D_C}$
q	number of buckles occurring in length of beam
m	number of bays in length of beam
p	number of buckles occurring across width of beam
n	number of bays in width of beam
r, s	integers

DESCRIPTION OF STRUCTURAL CALCULATIONS

The calculations presented in this paper apply to any one cell of a wing structure of the type indicated in figure 1. The complete structure has a number of shear webs, as in a multiweb wing, and located midway between the shear webs are single longitudinal rows of posts connecting stringers on the cover skins. This wing construction retains approximately one-half the number of shear webs found in a conventional multiweb structure and has been referred to as the multipost-stiffened wing (ref. 1). With the proper combination of component stiffnesses, a line of stringers and posts can provide the same stability against buckling to the compression skin under bending loads as a full-depth solid web.

Idealized Structure and Loading Condition

For the purpose of these calculations, the multipost-stiffened structure has been idealized as a long beam containing five basic elements to which stiffnesses can be assigned. These elements consist of two cover plates, two longitudinal stringers, and an arbitrary number of posts. The cover plates, which may be of unequal stiffness, are assumed to receive simple support from the shear webs along the side edges of the plate. The stringers are of identical cross section and are located along the longitudinal center line of each cover plate. The posts are idealized as a series of deflectional springs of equal stiffness which connect the stringers at equally spaced intervals.

The beam is loaded by a pure bending moment. The compressive and tensile stresses in the two sides of the beam are assumed to be uniformly distributed over the plate and stringer cross sections. The stress in the tension side of the beam can then be defined in terms of the buckling stress for the compression side of the beam by the following relationship which expresses the equality of the two loadings:

$$\sigma_T(2bt_T + A) = \sigma_C(2bt_C + A) \quad (1a)$$

Equation (1a) can be rewritten in this form

$$\frac{b^2\sigma_T t_T}{\pi^2 D_T} \frac{D_T}{D_C} \left(1 + \frac{1}{2} \delta_T\right) = \frac{b^2\sigma_C t_C}{\pi^2 D_C} \left(1 + \frac{1}{2} \delta_C\right) \quad (1b)$$

When the buckling-stress coefficients k_T and k_C are substituted for $\frac{b^2\sigma_T t_T}{\pi^2 D_T}$ and $\frac{b^2\sigma_C t_C}{\pi^2 D_C}$, respectively, the value of k_T can be given in terms of k_C as follows:

$$k_T = k_C \frac{D_C}{D_T} \frac{1 + \frac{1}{2} \delta_C}{1 + \frac{1}{2} \delta_T} \quad (2)$$

Stability Criteria

A stability criterion has been derived in reference 2 which is applicable to a beam with any number of rows of posts and stringers. As given here, the criterion has been modified slightly to conform to the notation and parameters adopted in the present paper:

$$\frac{1}{\beta^4 T} + \sum_{s=-\infty}^{\infty} \frac{1}{\left(2s + \frac{q}{m}\right)^2 \left[\left(2s + \frac{q}{m}\right)^2 \gamma_C - \beta^2 \partial_C k_C \right]} + \frac{1}{\sum_{r=-\infty}^{\infty} \frac{1}{\left[\left(2s + \frac{q}{m}\right)^2 + \left(2r + \frac{p}{n}\right)^2 \beta^2 \right]^2 - \left(2s + \frac{q}{m}\right)^2 \beta^2 k_C}} +$$

$$\frac{D_C}{D_T} \sum_{s=-\infty}^{\infty} \frac{1}{\left(2s + \frac{q}{m}\right)^2 \left[\left(2s + \frac{q}{m}\right)^2 \gamma_T + \beta^2 \partial_T k_T \right]} + \frac{1}{\sum_{r=-\infty}^{\infty} \frac{1}{\left[\left(2s + \frac{q}{m}\right)^2 + \left(2r + \frac{p}{n}\right)^2 \beta^2 \right]^2 + \left(2s + \frac{q}{m}\right)^2 \beta^2 k_T}} = 0 \quad (3)$$

The criterion is for the general instability mode of buckling in which the posts and stringers deflect with the cover skins and thus may be used to determine the critical combinations of post and stringer stiffness for which longitudinal nodes will form along the stringers at buckling (i.e., local instability of the compression skin). The values of p/n and q/m determine the general instability mode for the beam under consideration. For the beam shown in figure 1, the general instability mode corresponds to $p = 1$ (one buckle across the width of the beam) and $n = 2$ (two bays wide). The value of q is the number of buckles which occur in a beam length of m post spaces and the ratio q/m must be varied until the maximum value of post stiffness is found which satisfies the stability criterion for given values of the other variables.

The summations over r in equation (3) permit an exact representation of the buckle deflections across the width of both covers of the beam and the summations over s are associated with the variation of buckle deflections along the beam length. As a consequence of the large number of terms to be summed, solutions to this equation are practical only with high-speed computing machines. Solutions to the equation were obtained by the use of the National Bureau of Standards Eastern Automatic Computer (SEAC). The computational procedure is indicated in the appendix.

A useful simplified form of the stability criterion is obtained if the s -summation is eliminated from equation (3), so that the deflections of the covers are restricted to a sinusoidal variation in the length

direction. The remaining terms of the equation represent a complete solution for the case in which the post spacing approaches zero, in effect, a uniform distribution of stiffness along the center lines of the plates. The simplified stability criterion may be written as follows:

$$\frac{1}{\left(\frac{\lambda}{b}\right)^4 T} + \frac{1}{\gamma_C - \left(\frac{\lambda}{b}\right)^2 \delta_C k_C} + \sum_{r=-\infty}^{\infty} \frac{1}{1 + \left(2r + \frac{p}{n}\right)^2 \left(\frac{\lambda}{b}\right)^2 - \left(\frac{\lambda}{b}\right)^2 k_C} + \frac{\frac{D_C}{D_T}}{\gamma_T + \left(\frac{\lambda}{b}\right)^2 \delta_T k_T} + \sum_{r=-\infty}^{\infty} \frac{1}{1 + \left(2r + \frac{p}{n}\right)^2 \left(\frac{\lambda}{b}\right)^2 + \left(\frac{\lambda}{b}\right)^2 k_T} = 0 \quad (4)$$

The preliminary results presented in reference 3 showed that, if several posts are contained in each buckle length, the posts may be considered to be uniformly distributed with the result that equation (4) is applicable. The part of the calculations for which equation (4) was used was made with an IBM Card-Programmed Calculator.

Structural Parameters

Calculations were performed with the following values of the parameters appearing in the stability equations:

$$\beta = \frac{l}{b} = 0, \frac{1}{2}, 1, 2$$

$$k_C = \frac{\sigma_C t_C b^2}{\pi^2 D_C} = 4, 3.5, 3$$

$$\frac{D_T}{D_C} = \left(\frac{t_T}{t_C} \right)^3 = \frac{1}{8}, 1, \infty$$

$$\delta_C = \frac{A}{bt_C} = 0, 0.2, 0.4$$

For each of the possible combinations of these parameters, a range of combinations of the post-stiffness parameter T and stringer-stiffness parameter γ_C was determined which satisfies the stability equations. The evaluation of the nondimensional parameters T and γ_C for structural members is discussed in a subsequent section.

The values of $\beta = 1/2, 1$, and 2 , where β is the ratio of post spacing to bay width, were chosen to correspond to aircraft structural proportions. The calculations made with $\beta = 0$ permitted a reduction in the amount of calculation needed for the cases with finite post spacing, as is shown subsequently.

The flexural-stiffness ratio of the cover skins D_T/D_C was taken to be a function only of the skin-thickness ratio t_T/t_C . Thus, elastic cover skins having the same Young's modulus and Poisson's ratio have been assumed. Values of D_T/D_C were chosen to correspond to a tension skin one-half as thick as the compression skin, to skins of equal thickness, and to the limiting case of an infinitely stiff tension skin. This limiting case is useful in that the values of T resulting from these calculations are independent of the tension skin stiffness and are thus applicable to the determination of the stiffnesses required of members such as transverse stiffeners and formers which are not attached to the tension skin. In addition, this case is applicable when the posts are inclined to form a Warren truss with the stringers. The value of T then is a measure of the required depthwise spring stiffness at the panel points of the truss.

With respect to the stress values achieved in the beam at buckling, a value of k_C equal to 4 means that the compression skin receives enough support to buckle with a longitudinal node along the line of posts and thus behaves as a simply supported plate of width b and thickness t_C . No further increase in buckling stress is possible without adding torsional restraints along the shear webs or along the line of posts. Inasmuch as it is not always necessary to develop a stress value corresponding to $k_C = 4$ at a given cross section of a wing, the combinations of stringer and post stiffnesses required to develop buckling-stress coefficients of $k_C = 3.5$ and 3 were also computed.

The parameter δ_C is the ratio of the cross-sectional area of the stringers A to the cross-sectional area of the compression skin bt_C and, thus, determines the proportion of the total compression-panel load transmitted by the stringer. Because the size of stringers required in combination with posts to achieve a given buckling stress is considerably smaller than the stringers required for a conventionally stiffened sheet, values of $\delta_C = 0, 0.2, \text{ and } 0.4$ were chosen to cover the design range.

Other parameters appearing in the stability equations are δ_T and γ_T which may be defined in terms of the corresponding parameters δ_C and γ_C as follows:

$$\delta_T = \frac{A}{bt_T} = \frac{A}{bt_C} \frac{t_C}{t_T} = \delta_C \frac{t_C}{t_T} \quad (5)$$

and

$$\gamma_T = \frac{EI}{bD_T} = \frac{EI}{bD_C} \frac{D_C}{D_T} = \gamma_C \frac{D_C}{D_T} \quad (6)$$

RESULTS AND DISCUSSION

Interaction of Component Stiffnesses

The results of calculations made with the stability criteria and structural parameters discussed in the previous section are listed in tables I to III. For constant values of D_T/D_C , β , and δ_C , the combinations of γ_C and T required to achieve desired values of the buckling-stress coefficient k_C are given. The associated values of λ/b and q/m are also listed. When $\beta = 0$, buckling occurs with a sinusoidal variation in deflection in the length direction and from the ratio λ/b the natural buckle length for each combination of γ_C and T can be determined. The buckle pattern is more complex, however, when the posts are at discrete intervals. The distance between transverse nodes may be expected to be nonuniform and therefore no buckle length λ can be assigned. The number of buckles q occurring in m post spaces, however, was found in the solutions of the stability criterion (eq. (3)) and these values are listed in the tables as the ratio q/m .

The stiffness data in the tables are presented in design-chart form in figures 2 to 10. Each figure in this group is basically similar in that for constant values of D_T/D_C and δ_C a series of curves is presented which shows how values of the post-spacing ratio β equal to 2, 1, $1/2$, and 0 affect the combinations of γ_C and T required to achieve values of k_C equal to 4, 3.5, and 3. The data presented in figures 2 to 4 and figures 5 to 7 apply, respectively, to multipost-stiffened beams with equal-thickness tension and compression covers and with tension covers one-half as thick as the compression covers. Figures 8 to 10 apply to beams, such as those having Warren trusses, for which an assumption of an infinitely stiff tension cover is appropriate. In each of these groups of figures the effect of variations in the stringer area ratio δ_C is shown.

In order to serve as a guide to the presentation of data in figures 2 to 10, the curves in figure 2 have been identified with leaders and are discussed in detail. The three curves labeled $\beta = 0$ (posts at an infinitesimal spacing) give the combinations of the stringer-stiffness parameter γ_C and post-stiffness parameter T required to constrain the compression cover of a multipost-stiffened beam to buckle with stress coefficients equal to 4, 3.5, and 3. The maximum values of the stringer-stiffness parameter γ_C required to achieve the buckling-stress coefficients occur at the left end of these curves where the post-stiffness parameter T is zero. These maximum values of γ_C are those that would be computed for a compressed, infinitely long, simply supported plate with a single stringer (see ref. 4). As the stringer stiffness is decreased the post stiffness required to maintain the same stability increases. The data presented in reference 5 indicate that γ_C could be reduced to zero and buckling-stress coefficients of 3 and 3.5 could be achieved with very closely spaced posts of finite stiffness. However, a buckling-stress coefficient of 4 cannot be attained without the presence of a stringer, regardless of the magnitude of the post stiffness. Thus, for the curve associated with $\beta = 0$ and $k_C = 4$, the value of T approaches infinity for some value of γ_C between 0 and 0.7 as indicated in figure 2.

The combinations of γ_C and T associated with post-spacing ratios β equal to $1/2$, 1, and 2 lie along the $\beta = 0$ curves for a large part of their lengths and have been identified separately only along the parts where they deviate. The reason for the coincidence of these curves with the $\beta = 0$ curves in certain regions lies in the definition of the post-stiffness parameter T . The parameter T is a distributed stiffness; that is, it involves the stiffness of individual posts divided by the post spacing. Thus, when the buckle lengths are long enough to encompass several post spaces, the structural behavior is essentially the same as if a uniform distribution of post stiffness

($\beta = 0$) were present. As γ_C decreases, however, the buckle lengths become shorter relative to the post spaces (see the ratios of q/m in tables I to III) and the effect is to increase the value of T required for a given value of γ_C to achieve the same stability.

For the post-spacing ratios $\beta = 1$ and 2, values for γ_C were determined (dashed-line values in figs. 2 to 10) which would be required to achieve buckling-stress coefficients of 4, 3.5, and 3 when buckling occurs with transverse nodes through the post locations. In order for this mode of buckling to occur, the posts and the tension plate upon which the posts are supported must have sufficient stiffness to behave as if they were effectively rigid or nondeflecting. The precise points at which the curves for β equal to 1 and 2 intersect the dashed lines were not determined in these calculations, but an intersection at a finite value of T is indicated by the trend of the curves. Similar intersections of the $\beta = 1/2$ curves with the dashed lines are not indicated in any of the figures because at this post spacing the dashed-line values of γ_C are either very small or are negative. It is evident from the data that curves for other post-spacing ratios such as β equal to $3/4$ and $3/2$ could be established with reasonable accuracy by determining the dashed-line values of γ_C from a source such as reference 4 and fairing transitions into the $\beta = 0$ curves.

In a multipost-stiffened beam the stiffness of the tension cover influences the required stiffness of the posts and stringers. The magnitude of this effect is shown in figure 11. The post-spacing ratio β , the stringer area ratio δ_C , and the buckling-stress coefficient k_C have been held constant in this comparison. As would be expected, when the stiffness of the posts connecting the two covers is small, the stiffness of the tension cover has little effect on the stability of the compression cover. For more normal combinations of stringer and post stiffness, however, the tension-cover stiffness has an appreciable effect on the structural behavior. As indicated by the $D_T/D_C = 1/8$ curve (tension cover one-half as thick as compression cover), a buckling-stress coefficient of 4, corresponding to the formation of a longitudinal node along the stringer and post line, cannot be achieved even with infinitely stiff posts if the stringer-stiffness ratio γ_C is reduced to a value slightly less than 3. With $D_T/D_C = \infty$, stability could be maintained with $\gamma_C = 0$ and a finite value of T .

The calculations with $D_T/D_C = \infty$ were made not only to establish the extent to which the assumption of an infinitely stiff tension cover would influence the required values of post stiffness, as shown in figure 11, but also to establish the stiffness required of other types of supports which are not attached to the tension cover or for which the assumption of an infinitely stiff tension cover is appropriate. Such

supports may be in the form of transverse stiffeners or formers which can provide the equivalent of post stiffness by virtue of their beam bending stiffness. The replacement of one or more vertical posts by such transverse members may be desirable at fuel-cell and landing-gear locations in a multipost-stiffened wing. In this case the required beam stiffness of the transverse members may be determined from the values of T given in figures 8 to 10. The quantity F in the post-stiffness parameter $T = \frac{F}{l} \frac{b^3}{\pi^4 D_C}$ is taken to be the force applied at

the midspan of the transverse member required to produce a unit deflection of the member. If the support is a Warren truss, the value of F is the force required to produce a unit depthwise displacement at a panel point of the truss with the far ends of the diagonals entering the panel point prevented from displacing. Some practical considerations that arise in the evaluation of F for transverse beams and Warren trusses, as well as vertical posts, are discussed in the next section.

Evaluation of Post and Stringer Stiffness

The use of the post-stiffness parameter T in the present paper is a departure from the notation used in previous NACA papers dealing with post construction. The reason for this departure is that T appears to be a natural parameter as evidenced by the superposition of the data for various values of β in figures 2 to 10.

In order to clarify the meaning of T with respect to previous definitions of post stiffness, the following relationships are given. The relationship between T and the post-stiffness parameter S of references 2 and 3 is

$$T = \frac{Fb^3}{l\pi^4 D_C} = \frac{b}{l} \frac{Fb^2}{\pi^4 D_C} = \frac{S}{\beta} \quad (7)$$

The symbol S was also used to denote post stiffness in reference 5. The relationship between T and S of reference 5 (when one row of posts is used) is

$$T = \frac{S}{4\pi^2 \beta} \quad (8)$$

The evaluation of the quantities which determine T for a given construction is straightforward with the exception of the quantity F . Simple tension and compression tests made on specimens such as the one shown in figure 12(a) indicate that the flexibility of the attachments of posts to stringers and stringers to cover skins may easily have a greater influence on the effective value of F than the axial stiffness of the post itself. The effective values of F determined from experimental load-shortening curves indicated that from 15 to 30 percent of the theoretical axial stiffness of the posts alone was being realized. Even with this reduction in effective post stiffness due to attachment flexibility, however, the required values of T should be easily achieved with members of practical size.

A determination of the effective value of F for the panel points of a truss can be carried out in a loading test like that shown in figure 12(b). The load is applied to the chord members and the lower panel points are prevented from displacing. The displacement of the top chord is measured.

If the stiffness F is being provided by transverse members spanning the width between shear webs, the quantity F will be in the form of a bending stiffness. For the case of a member with simply supported ends and a rigid connection at the midspan to the longitudinal stringer, the value of F is equal to $\frac{6(EI)_{\text{trans}}}{b^3}$ (where $(EI)_{\text{trans}}$ is the flexural stiffness of the transverse member). A suitable reduction in this value should be made to account for a flexible connection between the longitudinal stringer and the transverse member.

For the calculation of the stringer stiffness parameter γ_C , approximately correct values should be obtained if the stringer cross section is sturdy and the moment of inertia of the stringer is calculated about the plane of attachment to the cover skins. This procedure can be shown to be justified when δ_C and γ_C are small, as is likely to be the case with multipost construction.

Numerical Comparison

The effect of using stringers in conjunction with posts in a multipost-stiffened structure can be illustrated by comparing the present results with those obtained in reference 5 which apply to a multipost-stiffened beam without stringers. For example, consider a beam with equal-stiffness covers $D_T/D_C = 1$ and a post-spacing ratio of $\beta = 1$. If such a beam were constructed without stringers along the row of posts, the maximum buckling-stress coefficient that could be

achieved would be $k_G = 1.56$ and the required post stiffness would correspond to $T = 0.62$. (From table I of ref. 5 and in the notation of that paper, $\beta = 0.50$, $k = 6.250$, and $S = 24.4$.) With these same values of post stiffness and post spacing, the effect of adding stringers to the beam can be determined from figures 2 to 4, depending upon the stringer area ratio δ_G selected. The values of γ_G required to raise the buckling-stress coefficient of the structure from $k_G = 1.56$ to $k_G = 3, 3.5$, and 4 have been read from these figures and plotted to form the curves presented in figure 13. These curves show that the addition of stringers of relatively small moment of inertia produces a large increase in the buckling-stress coefficient for the structure.

The increase in buckling-stress coefficient shown in figure 13 could also be achieved in a beam with posts alone by either adding more rows or decreasing the longitudinal spacing, or both. The merit of this alternative as opposed to adding stringers must be decided on the basis of weight and fabrication problems for the structure in question.

CONCLUDING REMARKS

The data presented give numerical values of nondimensional stiffness quantities needed for the design of the components of a multipost-stiffened wing. Procedures that might be used in evaluating these stiffness quantities for practical construction have been indicated. The results indicate that, over a wide range of structural parameters, the required stiffnesses can be achieved with structural members of practical size.

A comparison of these results with those of a similar analysis of multipost-stiffened beams without stringers shows that, for the same post configuration, appreciably higher buckling stresses can be developed when small stringers are used in conjunction with posts. The addition of stringers makes possible the development of buckling-stress values in the compression surface corresponding to simple support along the lines of posts.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., October 7, 1953.

APPENDIX

COMPUTATIONAL PROCEDURE

The numerical data presented in tables I to III were computed by the use of the stability criteria given previously as equations (3) and (4). The data for the cases in which $\beta = 1/2, 1$, and 2 were obtained from equation (3) which was coded for solution on the National Bureau of Standards Eastern Automatic Computer (SEAC). The $\beta = 0$ case was computed by the use of equation (4) coded for solution on an IBM Card-Programmed Calculator.

The procedure used with equation (3) to calculate a given point listed in tables I to III may be summarized as follows:

(1) Insert the desired values of the structural parameters into the code for equation (3).

(2) In order to calculate the value of T associated with a given value of γ_C , assume a value of q/m and sum the terms in equation (3) until a value of T is computed to the desired accuracy.

(3) For the given value of γ_C , vary q/m in small steps and sum the series for each variation in q/m until a maximum value of T is obtained.

The same general procedure was also used in solving equation (4) with the exception that, for given values of the structural parameters, T was maximized with respect to λ/b . With respect to the accuracy of the results, the number of terms summed in the stability criterion were adjusted until it was evident that the error in the value of T would be less than 1 percent.

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TABLE I.- DATA COMPUTED FROM STABILITY EQUATION

$$[k_c = 4.0]$$

δ_c	β	$\frac{D_T}{D_C} = \frac{1}{8}$				$\frac{D_T}{D_C} = 1$				$\frac{D_T}{D_C} = \infty$			
		γ_c	λ/b	q/m	T	γ_c	λ/b	q/m	T	γ_c	λ/b	q/m	T
0	0	48.70	5.24	---	0	48.70	5.24	---	0	48.70	5.24	---	0
		40	4.80	---	.0150	40	4.80	---	.0146	40	4.79	---	.0140
		30	4.12	---	.0480	30	4.15	---	.0457	30	4.12	---	.0412
		20	3.48	---	.1268	20	3.39	---	.1170	10	2.22	---	.2925
		10	3.04	---	.4390	10	2.40	---	.3862	6	1.60	---	.6268
		8	2.38	---	.7069	6	1.78	---	.8777	4	1.27	---	1.1384
		5	2.13	---	1.8370	2	1.20	---	5.1134	2	1.03	---	2.4335
		3.5	2.12	---	5.6371	1	1.16	---	16.727	1	.933	---	3.5964
		3	2.12	---	16.654	.7	1.16	---	42.540	.4	.873	---	4.4883
	1/2	8	---	0.21	.7081	5	---	0.31	1.1878	4	---	0.40	1.1449
		4	---	.24	3.4221	3	---	.39	2.7784	2	---	.49	2.4528
		3	---	.24	18.553	2	---	.42	5.3720	1	---	.55	3.7314
	1	10	---	.38	.4779	10	---	.42	.3922	10	---	.46	.2943
		8	---	.42	.7315	8	---	.48	.5692	8	---	.52	.4127
		6	---	.46	1.3462	5	---	.66	1.4138	5	---	.74	.9216
		4.5	---	.50	3.1738	4.5	---	.78	1.9987	4.5	---	.84	1.2455
	2	25	---	.53	.0800	25	---	.53	.0745	25	---	.54	.0639
		20	---	.60	.1367	18	---	.64	.1583	20	---	.62	.1014
		15	---	.72	.3177	15	---	.76	.2732	15	---	.78	.1955
.2	0	73.41	5.83	---	0	73.41	5.83	---	0	73.41	5.83	---	0
		60	5.31	---	.0151	60	5.37	---	.0158	60	5.30	---	.0142
		40	4.41	---	.0635	40	4.38	---	.0607	40	4.31	---	.0536
		20	3.35	---	.2525	20	3.13	---	.2307	20	2.96	---	.1808
		10	2.37	---	.7548	10	2.22	---	.6660	10	2.00	---	.4782
		6	1.91	---	1.7144	5	1.52	---	1.8976	4	1.22	---	1.7159
		3	1.68	---	6.5390	1.5	1.18	---	11.586	2	.996	---	3.2111
		2.5	1.68	---	11.056	1	1.17	---	25.752	1	.906	---	4.5408
		2	1.68	---	34.533	.8	1.17	---	40.142	.7	.873	---	5.0195
	1/2	5	---	.28	2.3615	5	---	.33	1.9108	4	---	.42	1.6888
		4	---	.30	3.5794	3	---	.40	4.2496	2	---	.51	3.2936
		3	---	.30	6.8284	2	---	.42	8.0732	1	---	.57	4.0524
	1	10	---	.42	.7783	15	---	.36	.3651	10	---	.50	.4842
		7	---	.50	1.4626	8	---	.52	.9820	8	---	.58	.6709
		6	---	.56	2.0244	6	---	.64	1.6956	6	---	.72	1.0854
		5.3	---	.70	2.8697	5.3	---	.78	2.4726	5.3	---	.84	1.5214
	2	25	---	.57	.1816	25	---	.58	.1698	30	---	.56	.0962
		22	---	.61	.2399	22	---	.63	.2222	22	---	.66	.1686
		19	---	.69	.3542	19	---	.71	.3240	18	---	.80	.2843
.4	0	103.25	6.36	---	0	103.25	6.36	---	0	103.25	6.36	---	0
		90	5.98	---	.0096	90	5.98	---	.0095	90	5.98	---	.0092
		60	5.00	---	.0520	60	4.90	---	.0514	60	4.87	---	.0459
		30	3.65	---	.2126	30	3.59	---	.2027	30	3.58	---	.1614
		10	2.21	---	1.1052	10	1.88	---	.9953	10	1.82	---	.7037
		4	1.50	---	4.4587	4	1.34	---	3.8421	4	1.14	---	2.2971
		3	1.43	---	7.3721	2	1.19	---	10.305	2	.952	---	4.0539
		2	1.43	---	17.990	1	1.18	---	32.342	1	.874	---	5.5507
		1.5	1.43	---	58.839	.8	1.18	---	54.514	.4	.814	---	6.8425
	1/2	10	---	.23	1.1073	5	---	.34	2.7987	2	---	.53	4.2124
		3	---	.35	7.7664	2	---	.43	11.604	1	---	.59	6.1342
		2	---	.35	21.951	1	---	.44	87.382	.4	---	.67	9.6624
	1	8	---	.52	1.6915	8	---	.56	1.5379	8	---	.64	.9986
		6.5	---	.64	2.6102	6.5	---	.68	2.3729	6.5	---	.76	1.4644
	2	6.05	---	.80	3.3468	6.05	---	.82	3.1100	6.05	---	.86	1.8963
		30	---	.56	.2284	30	---	.56	.2173	30	---	.60	.1667
		25	---	.64	.3311	25	---	.64	.3142	25	---	.68	.2285
		21	---	.80	.5971	21	---	.80	.5642	21	---	.86	.3813

TABLE II.- DATA COMPUTED FROM STABILITY EQUATION

$$[k_C = 3.5]$$

α_C	β	$\frac{D_T}{D_C} = \frac{1}{8}$				$\frac{D_T}{D_C} = 1$				$\frac{D_T}{D_C} = \infty$			
		γ_C	λ/b	q/m	T	γ_C	λ/b	q/m	T	γ_C	λ/b	q/m	T
0	0	35.50	4.85	---	0	35.50	4.85	---	0	35.50	4.85	---	0
		30	4.51	---	.0125	30	4.51	---	.0123	30	4.50	---	.0119
		20	3.75	---	.0604	20	3.71	---	.0569	20	3.67	---	.0505
		10	2.76	---	.2529	10	2.64	---	.2191	10	2.50	---	.1748
		8	2.52	---	.3739	6	2.07	---	.4804	6	1.90	---	.3591
		5	2.17	---	.8435	3	1.53	---	1.2520	3	1.36	---	.8282
		3	1.99	---	2.3443	1.5	1.29	---	2.8354	1.5	1.12	---	1.5244
		2.5	1.98	---	3.8873	.8	1.21	---	5.2054	.8	1.01	---	2.1039
		1.8	1.98	---	37.430	.4	1.16	---	9.2436	.4	.938	---	2.6006
	1/2	4	---	0.24	1.2784	4	---	0.29	.8545	3	---	0.37	.8325
		3	---	.25	2.3785	2	---	.37	2.1038	1.5	---	.45	1.5387
		2.5	---	.25	3.9978	1	---	.41	4.6055	.8	---	.51	2.1717
	1	2	---	.25	12.000	.4	---	.43	15.412	.4	---	.54	2.7450
		6	---	.44	.6403	6	---	.48	.4955	6	---	.54	.3608
		4	---	.50	1.4853	4	---	.60	.9634	4.5	---	.62	.5378
	2	3.5	---	.52	2.1334	3.5	---	.66	1.2474	3.5	---	.72	.7828
		3	---	.55	3.7789	3	---	.78	1.9124	3	---	.82	1.0968
.2	0	20	---	.56	.0610	20	---	.56	.0577	20	---	.56	.0510
		14	---	.64	.1525	14	---	.68	.1377	15	---	.66	.0935
		12	---	.76	.2621	12	---	.76	.2264	12	---	.79	.1692
		54.09	5.41	---	0	54.09	5.41	---	0	54.09	5.41	---	0
		40	4.71	---	.0241	40	4.70	---	.0236	40	4.68	---	.0221
		30	4.13	---	.0602	30	4.11	---	.0580	30	4.06	---	.0513
		10	2.53	---	.4462	20	3.38	---	.1346	10	2.26	---	.3010
		6	2.08	---	.9551	10	2.45	---	.4012	6	1.72	---	.5739
	1/2	3	1.72	---	2.7139	6	1.92	---	.8162	2.5	1.20	---	1.4689
		2	1.65	---	5.5136	2.5	1.36	---	2.5127	1.5	1.05	---	2.1514
		1.5	1.64	---	10.762	1	1.22	---	6.3811	.8	---	.948	2.8861
	1	1.2	1.63	---	24.532	.4	1.17	---	13.330	.4	---	.896	3.4309
.4	0	4	---	.27	1.7355	3	---	.34	2.0240	4	---	.36	.9235
		1.5	---	.31	12.347	1	---	.42	7.3777	2	---	.45	1.7789
		1.3	---	.31	22.468	.6	---	.43	14.603	1	---	.52	2.7088
		1.2	---	.31	38.221	.4	---	.44	33.552	.4	---	.57	3.7962
	1	10	---	.39	.4528	10	---	.42	.4063	10	---	.44	.3017
		5	---	.53	1.4200	6	---	.52	.8712	6	---	.58	.5919
		4	---	.61	2.3114	4	---	.68	1.8310	4.5	---	.70	.8879
	2	3.65	---	.74	3.1459	3.7	---	.78	2.3532	3.7	---	.82	1.3444
		25	---	.52	.0939	25	---	.52	.0891	25	---	.56	.0758
		18	---	.64	.1942	18	---	.64	.1810	18	---	.67	.1418
	2	15	---	.72	.3296	15	---	.76	.3022	15	---	.78	.2180
.4	0	76.60	5.91	---	0	76.60	5.91	---	0	76.60	5.91	---	0
		60	5.29	---	.0186	60	5.28	---	.0184	60	5.27	---	.0174
		40	4.40	---	.0680	40	4.37	---	.0660	40	4.29	---	.0575
		20	3.24	---	.2474	20	3.17	---	.2348	20	2.99	---	.1834
		10	2.40	---	.6829	10	2.30	---	.6335	8	1.84	---	.5940
		6	1.94	---	1.3748	6	1.84	---	1.2400	4	1.33	---	1.3036
		3	1.57	---	3.5069	2	1.29	---	4.5498	2	1.07	---	2.3473
		2	1.48	---	6.1920	1	1.20	---	8.9242	1	.938	---	3.4216
	1/2	.8	1.44	---	39.203	.4	1.17	---	18.502	.4	.844	---	4.4146
		4	---	.30	2.4035	3	---	.36	2.9609	2	---	.47	2.3926
		2	---	.34	6.5961	1	---	.42	10.728	1	---	.54	3.5670
	1	1.5	---	.35	11.011	.4	---	.45	57.838	.6	---	.58	4.4264
		1	---	.35	34.083					.4	---	.61	5.1682
	2	6	---	.54	1.5497	6	---	.56	1.3864	8	---	.54	.6063
		4.35	---	.78	3.3526	4.35	---	.80	2.9880	5	---	.72	1.1699
										4.35	---	.86	1.6947
.4	2	30	---	.52	.1282	30	---	.52	.1230	30	---	.56	.1005
		20	---	.64	.2956	20	---	.68	.2784	20	---	.68	.2055
		17.5	---	.76	.4630	17.5	---	.80	.4370	17.5	---	.82	.3020

TABLE III.- DATA COMPUTED FROM STABILITY EQUATION

$$[k_c = 3.0]$$

δ_c	β	$\frac{D_T}{D_C} = \frac{1}{8}$				$\frac{D_T}{D_C} = 1$				$\frac{D_T}{D_C} = \infty$			
		γ_c	λ/b	q/m	T	γ_c	λ/b	q/m	T	γ_c	λ/b	q/m	T
0	0	24.54	4.42	---	0	24.54	4.42	---	0	24.54	4.42	---	0
		20	4.08	---	.0146	20	4.08	---	.0143	20	4.08	---	.0137
		15	3.58	---	.0453	15	3.56	---	.0430	15	3.53	---	.0389
		10	2.98	---	.1180	10	2.91	---	.1072	10	2.84	---	.0914
		5	2.29	---	.3965	5	2.15	---	.3220	5	2.03	---	.2488
		2	1.87	---	1.7712	2	1.54	---	.9734	2	1.39	---	.6538
		1.5	1.82	---	4.8086	1	1.33	---	1.7962	1	1.17	---	1.0481
		1.1	1.81	---	9.1695	.6	1.25	---	2.5041	.6	1.08	---	1.3094
		.94	1.80	---	33.005	.4	1.22	---	3.0560	.4	1.03	---	1.4875
	1/2	4	---	.023	.5642	3	---	.029	.6232	4	---	.028	.3252
		1.5	---	.28	3.3770	1.5	---	.33	1.3046	2	---	.36	.6563
		1.2	---	.28	6.9366	.8	---	.39	2.1942	1	---	.43	1.0622
	1	1	---	.28	23.408	.4	---	.42	3.3964	.4	---	.49	1.5270
		6	---	.41	.3002	6	---	.44	.2524	6	---	.46	.1976
		3	---	.51	1.0240	3	---	.58	.6908	3	---	.64	.4706
		2.2	---	.57	2.2217	2.2	---	.68	1.1646	2	---	.80	.8423
	2	1.9	---	.62	4.1418	1.9	---	.82	1.8183	1.9	---	.86	.9937
		15	---	.56	.0467	15	---	.56	.0431	15	---	.56	.0596
		10	---	.72	.1465	10	---	.72	.1309	10	---	.73	.1046
.2	0	9	---	.88	.2540	9	---	.88	.2203	9	---	.89	.1639
		37.61	4.95	---	0	37.61	4.95	---	0	37.61	4.95	---	0
		30	4.48	---	.0170	30	4.48	---	.0168	30	4.47	---	.0160
		20	3.73	---	.0657	20	3.71	---	.0631	20	3.66	---	.0555
		10	3.05	---	.2339	10	2.68	---	.2218	10	2.63	---	.1751
		6	2.26	---	.5138	6	2.16	---	.4266	6	2.00	---	.3361
		2	1.67	---	2.2010	2.5	1.55	---	1.2699	2.5	1.37	---	.8308
		1	1.56	---	5.7124	1.5	1.37	---	2.0573	1	1.09	---	1.5303
	1/2	.6	1.56	---	14.119	1	1.28	---	2.8250	.6	1.01	---	1.8787
		.4	1.56	---	52.320	.4	1.19	---	4.7137	.4	.962	---	2.0884
		6	---	.22	.5138	3	---	.30	1.0501	4	---	.31	.5294
		1.5	---	.31	3.3482	2	---	.34	1.6069	3	---	.34	.7070
	1	.7	---	.33	13.888	1	---	.39	2.9543	1	---	.46	1.5582
		.5	---	.33	78.720	.5	---	.42	4.7668	.4	---	.53	2.2071
		10	---	.36	.2823	8	---	.42	.3108	6	---	.50	.3406
	2	4	---	.52	.9954	4	---	.56	.8265	4	---	.62	.5552
		3	---	.60	1.6946	3	---	.64	1.3148	3	---	.72	.8049
		2.5	---	.77	2.9409	2.5	---	.82	2.2160	2.5	---	.86	1.2200
.4	0	18	---	.56	.0864	20	---	.56	.0641	18	---	.60	.0705
		14	---	.64	.1554	14	---	.64	.1445	14	---	.68	.1156
		12	---	.72	.2436	12	---	.76	.2238	12	---	.77	.1670
		53.88	5.42	---	0	53.88	5.42	---	0	53.88	5.42	---	0
		40	4.74	---	.0232	40	4.73	---	.0228	40	4.71	---	.0214
		20	3.48	---	.1328	20	3.41	---	.1322	20	3.33	---	.1068
		10	2.59	---	.3946	10	2.51	---	.3693	10	2.34	---	.2772
		6	2.11	---	.7818	6	2.03	---	.7138	6	1.82	---	.5053
	1/2	2	1.55	---	2.9068	4	1.73	---	1.1495	3	1.36	---	1.0085
		1	1.43	---	6.1708	2	1.41	---	2.3405	2	1.36	---	1.4046
		.6	1.40	---	10.657	1	1.26	---	4.0727	1	1.03	---	2.0874
		.4	1.38	---	16.579	.4	1.17	---	6.7247	.4	.911	---	2.8008
	1	4	---	.27	1.3064	3	---	.32	1.5866	2	---	.42	1.4071
		1	---	.35	6.9553	1	---	.40	4.4091	1	---	.49	2.1336
		.4	---	.37	43.054	.4	---	.44	8.8607	.6	---	.54	2.6503
	2	6	---	.48	.8294	6	---	.50	.7527	6	---	.56	.5177
		4	---	.58	1.5648	4	---	.60	1.5660	4	---	.68	.8502
		3.1	---	.80	3.0462	3.1	---	.82	2.6173	3.5	---	.74	1.0388
.6	0	20	---	.60	.1426	20	---	.60	.1368	20	---	.60	.1102
		16	---	.68	.2331	16	---	.68	.2211	16	---	.70	.1665
		14	---	.80	.3714	14	---	.80	.3497	14	---	.82	.2466
		53.88	5.42	---	0	53.88	5.42	---	0	53.88	5.42	---	0
		40	4.74	---	.0232	40	4.73	---	.0228	40	4.71	---	.0214
		20	3.48	---	.1328	20	3.41	---	.1322	20	3.33	---	.1068
		10	2.59	---	.3946	10	2.51	---	.3693	10	2.34	---	.2772
		6	2.11	---	.7818	6	2.03	---	.7138	6	1.82	---	.5053
	1/2	2	1.55	---	2.9068	4	1.73	---	1.1495	3	1.36	---	1.0085
		1	1.43	---	6.1708	2	1.41	---	2.3405	2	1.36	---	1.4046
		.6	1.40	---	10.657	1	1.26	---	4.0727	1	1.03	---	2.0874
		.4	1.38	---	16.579	.4	1.17	---	6.7247	.4	.911	---	2.8008
	1	4	---	.27	1.3064	3	---	.32	1.5866	2	---	.42	1.4071
		1	---	.35	6.9553	1	---	.40	4.4091	1	---	.49	2.1336
		.4	---	.37	43.054	.4	---	.44	8.8607	.6	---	.54	2.6503
	2	6	---	.48	.8294	6	---	.50	.7527	6	---	.56	.5177
		4	---	.58	1.5648	4	---	.60	1.5660	4	---	.68	.8502
		3.1	---	.80	3.0462	3.1	---	.82	2.6173	3.5	---	.74	1.0388

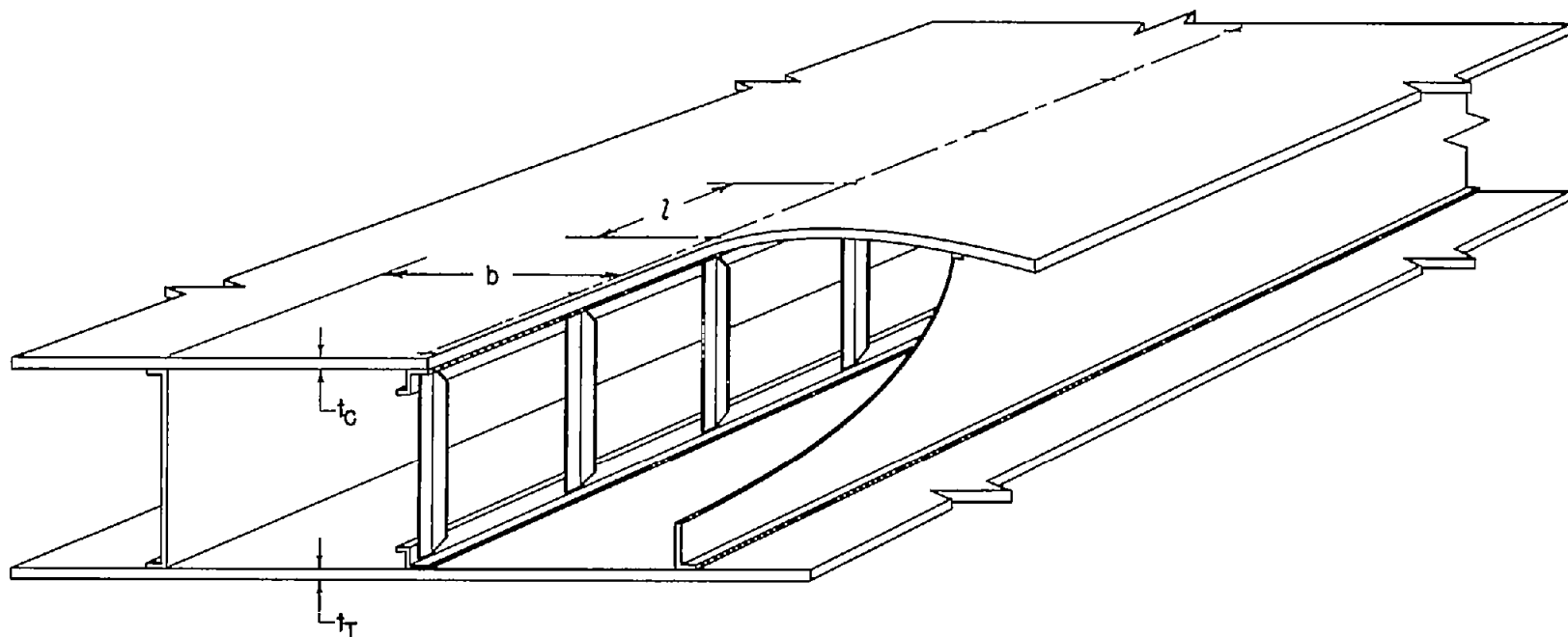


Figure 1.—Portion of multipost-stiffened beam.

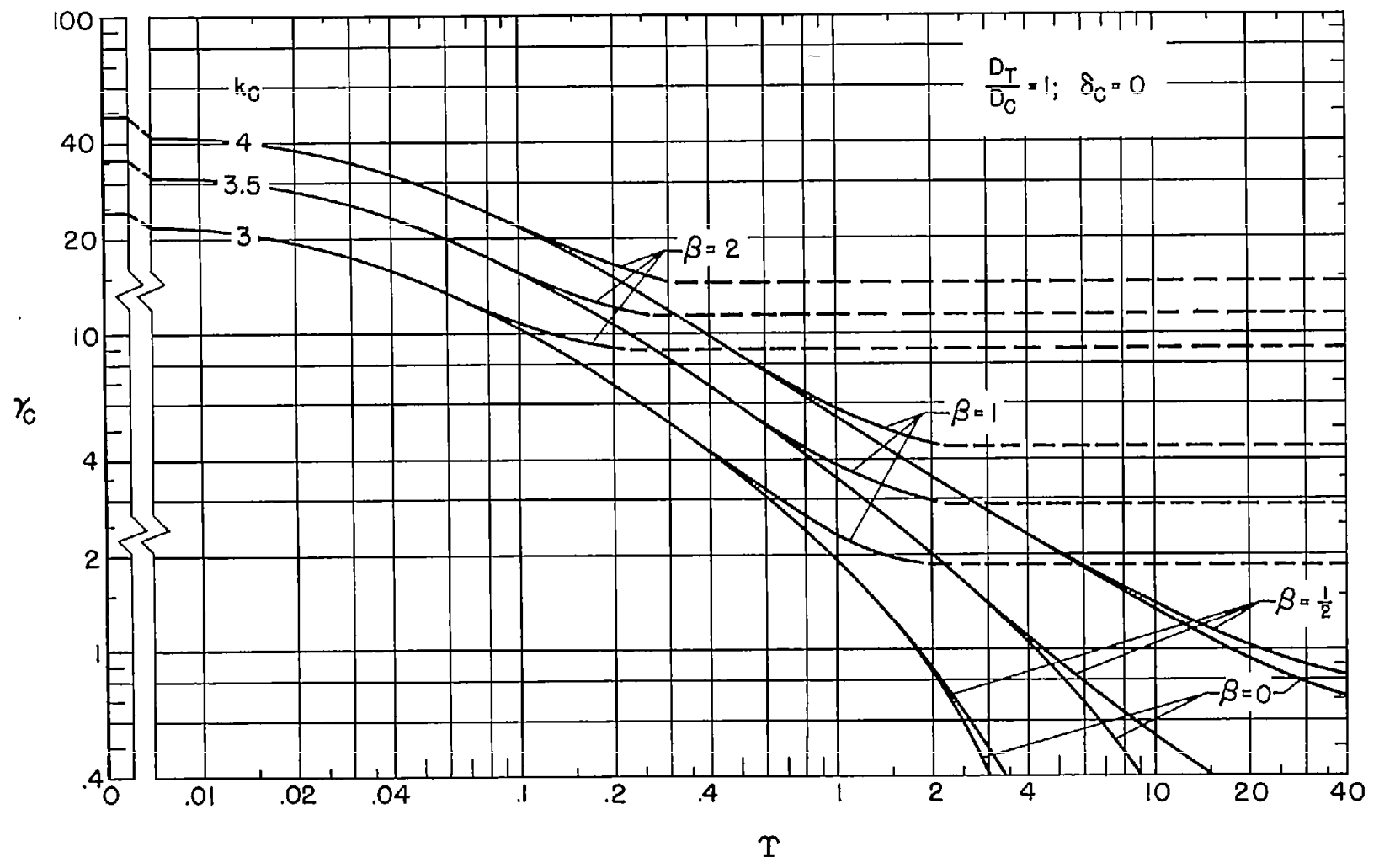


Figure 2.— Interaction of stringer stiffness and post stiffness. $\frac{D_T}{D_C} = 1$; $\delta_C = 0$.

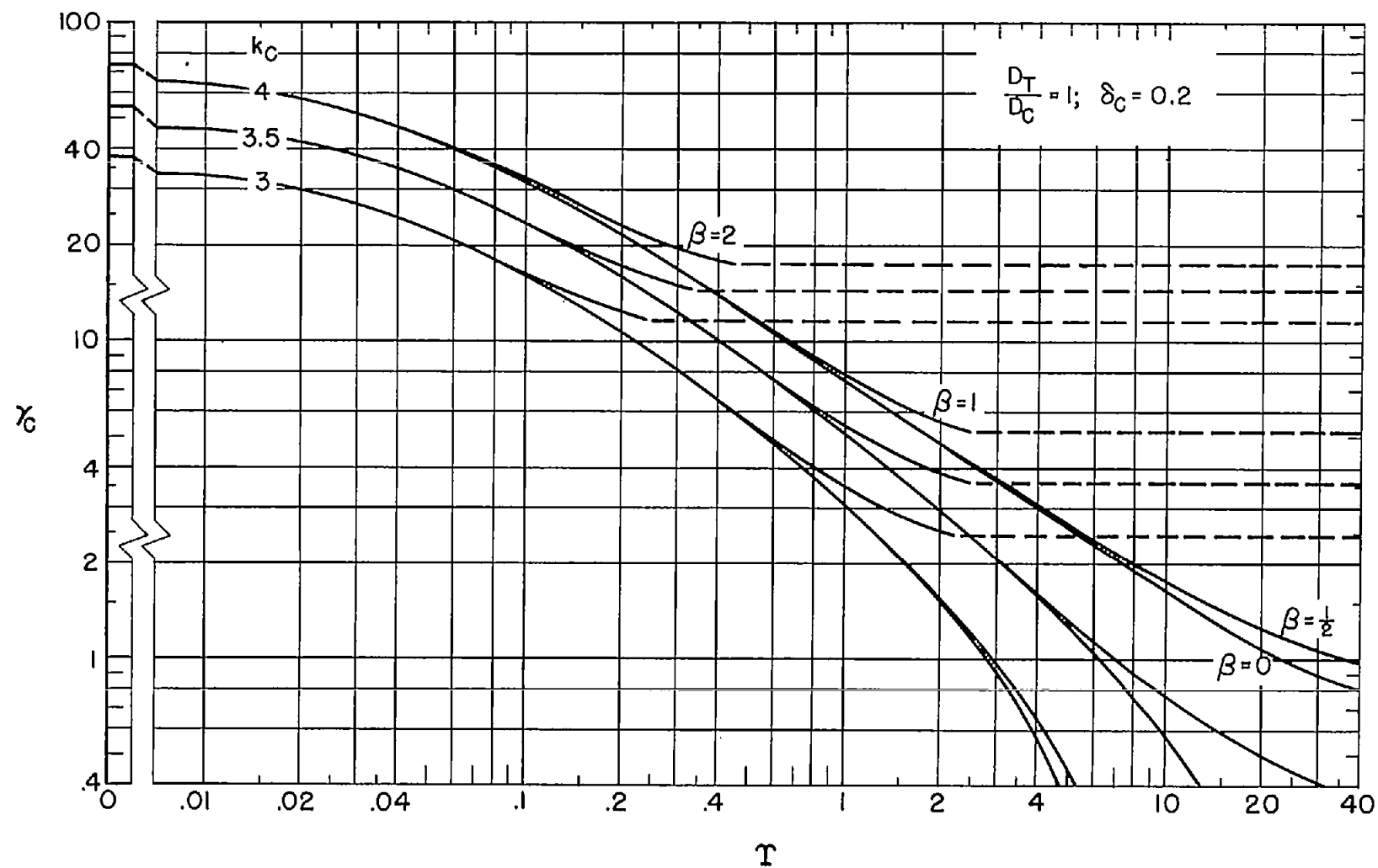


Figure 3.— Interaction of stringer stiffness and post stiffness. $\frac{D_T}{D_C} = 1$; $\delta_C = 0.2$.

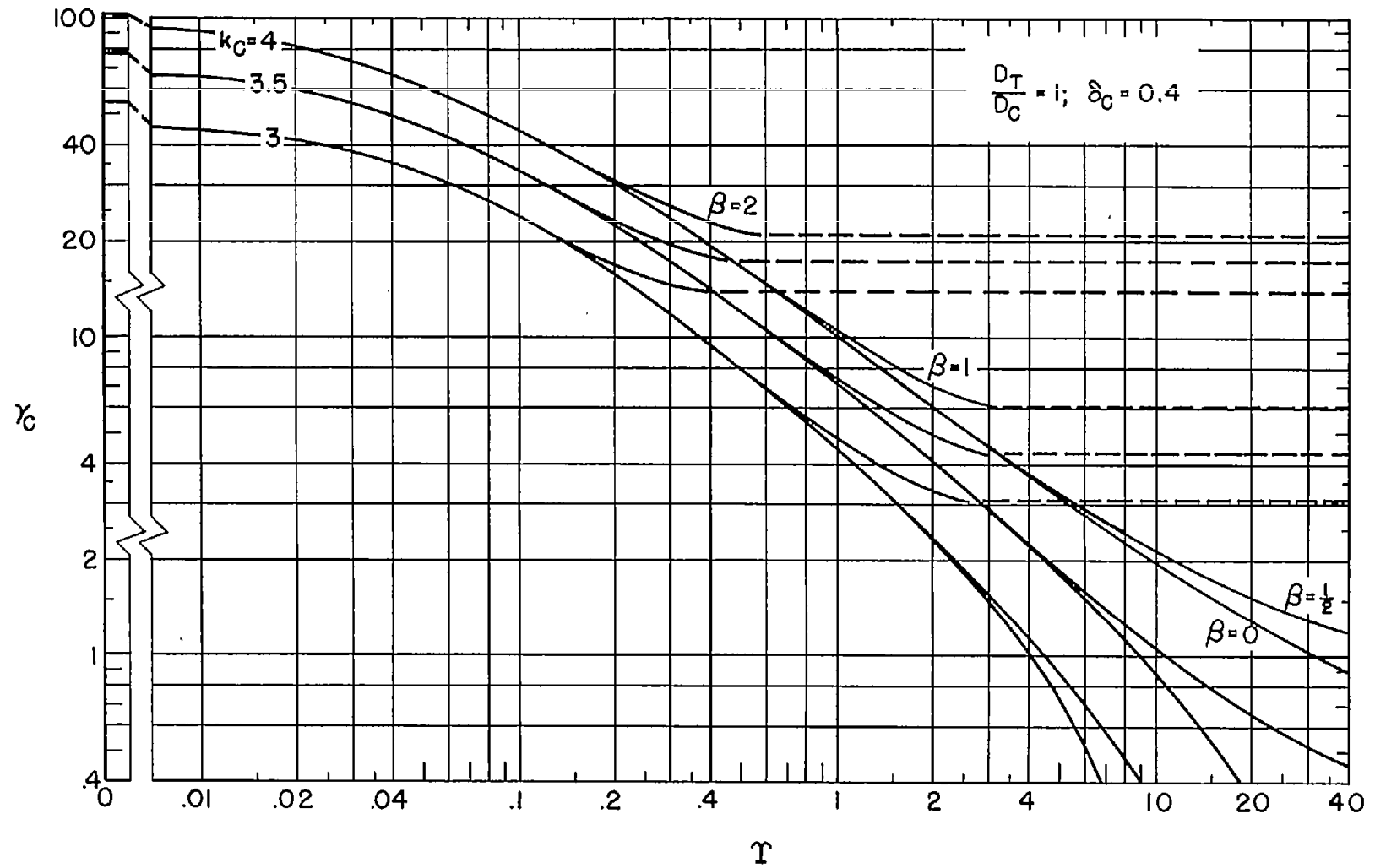


Figure 4.—Interaction of stringer stiffness and post stiffness. $\frac{D_T}{D_C} = 1$; $\delta_C = 0.4$.

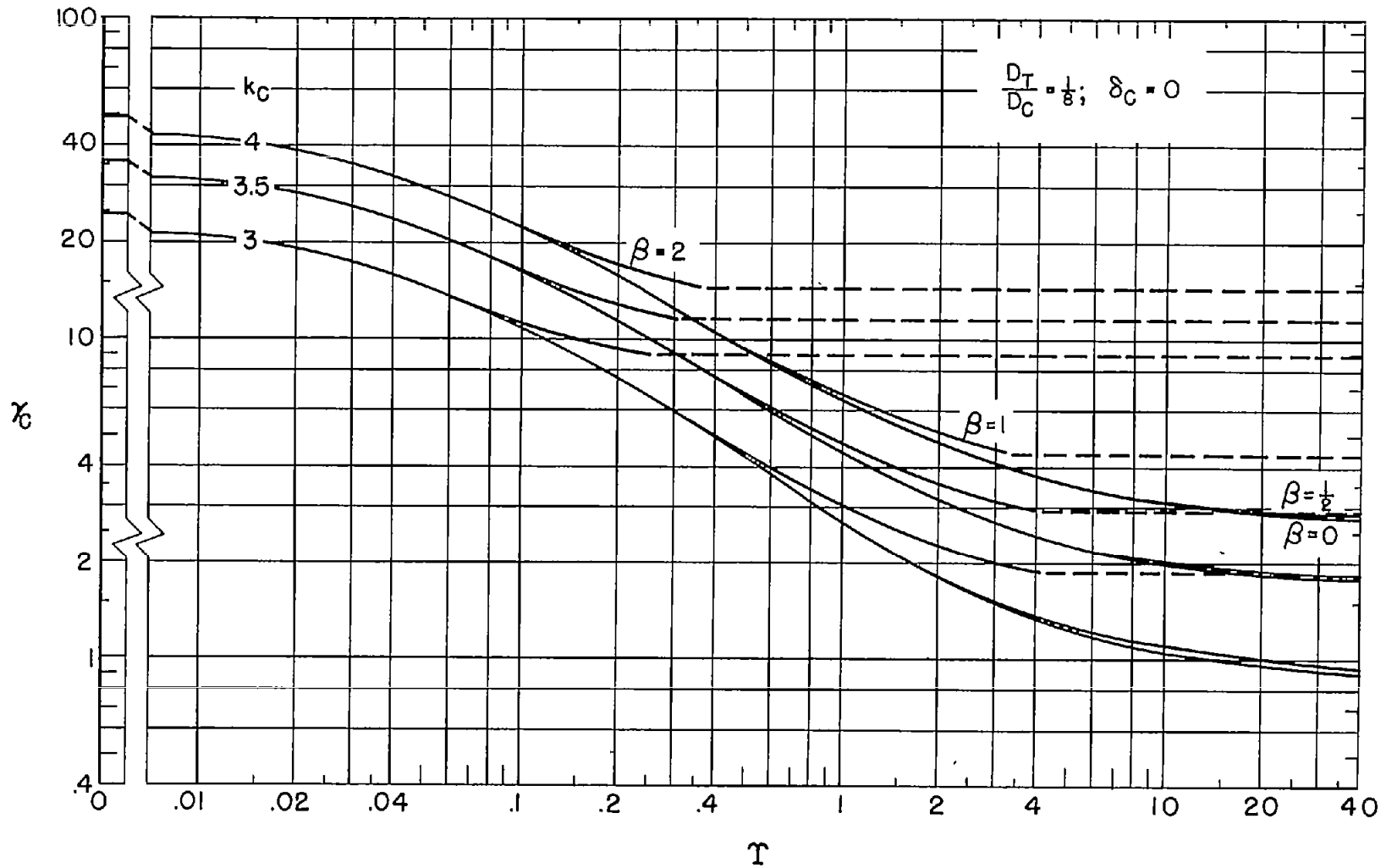


Figure 5.—Interaction of stringer stiffness and post stiffness. $\frac{D_I}{D_C} = \frac{1}{8}; \delta_C = 0$.

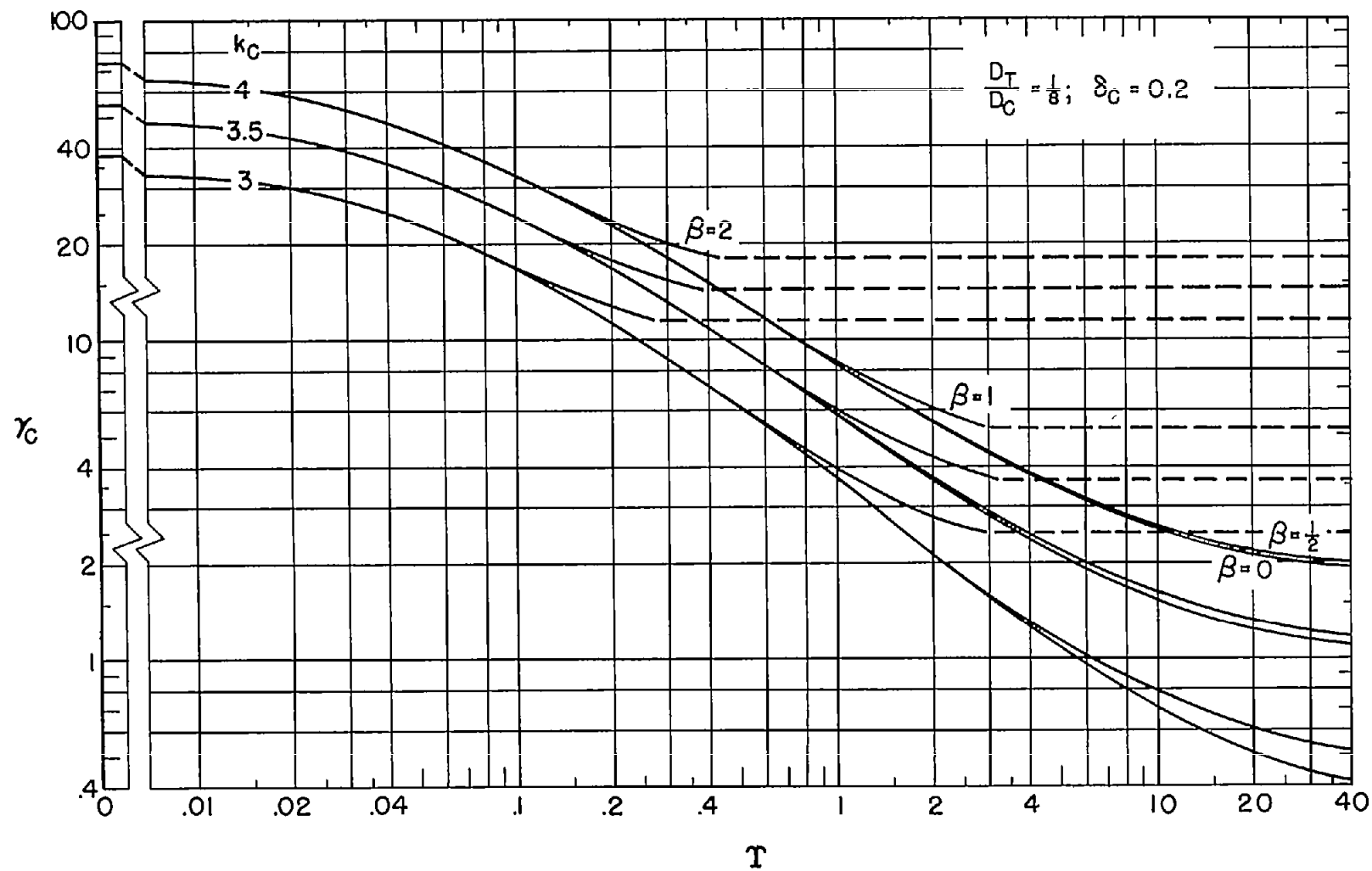


Figure 6.—Interaction of stringer stiffness and post stiffness. $\frac{D_T}{D_C} = \frac{1}{8}$; $\delta_C = 0.2$.

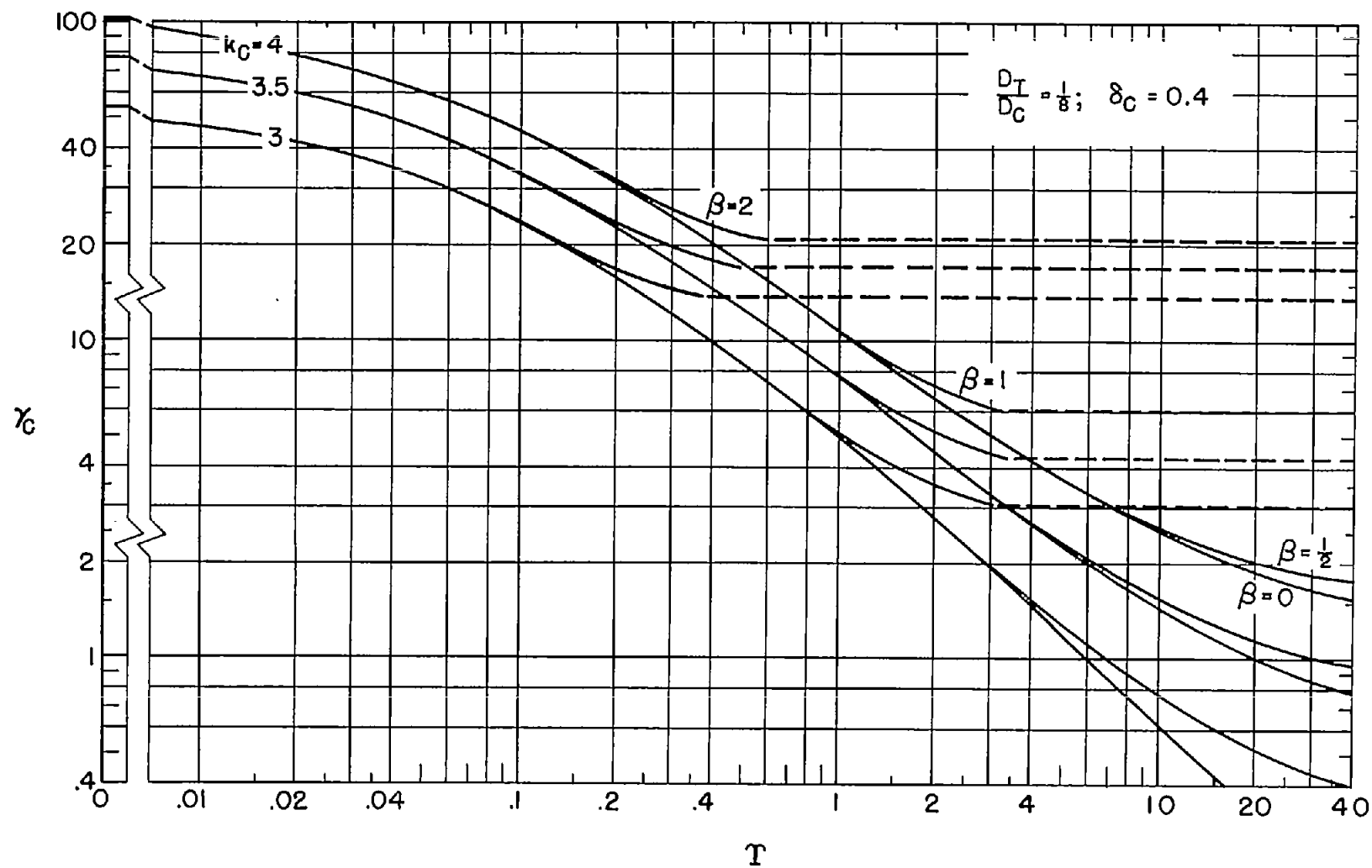


Figure 7.—Interaction of stringer stiffness and post stiffness. $\frac{D_I}{D_C} = \frac{1}{8}$; $\delta_C = 0.4$.

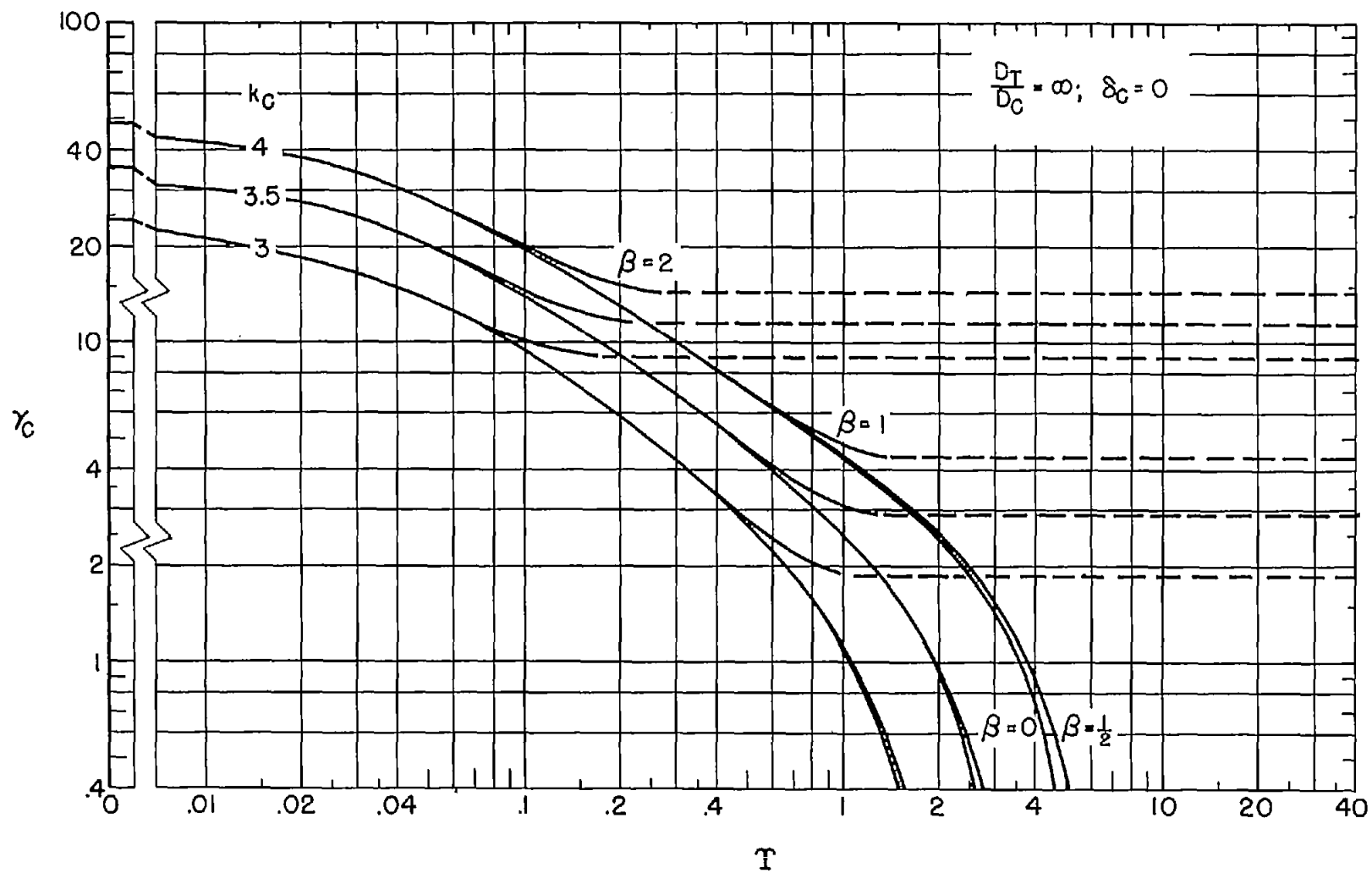


Figure 8.—Interaction of stringer stiffness and post stiffness. $\frac{D_T}{D_C} = \infty$; $\delta_C = 0$.

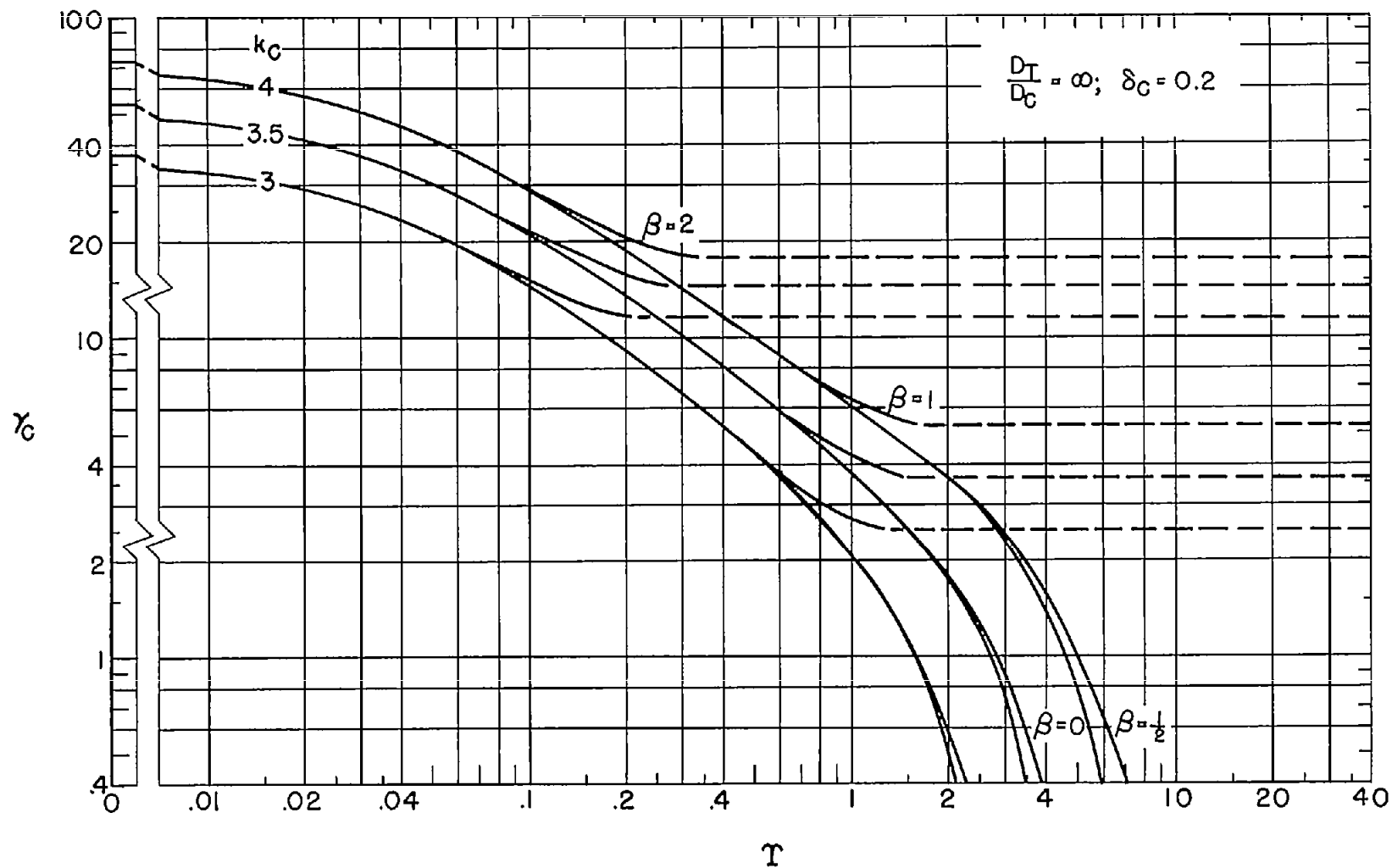


Figure 9.—Interaction of stringer stiffness and post stiffness. $\frac{D_T}{D_C} = \infty$; $\delta_C = 0.2$.

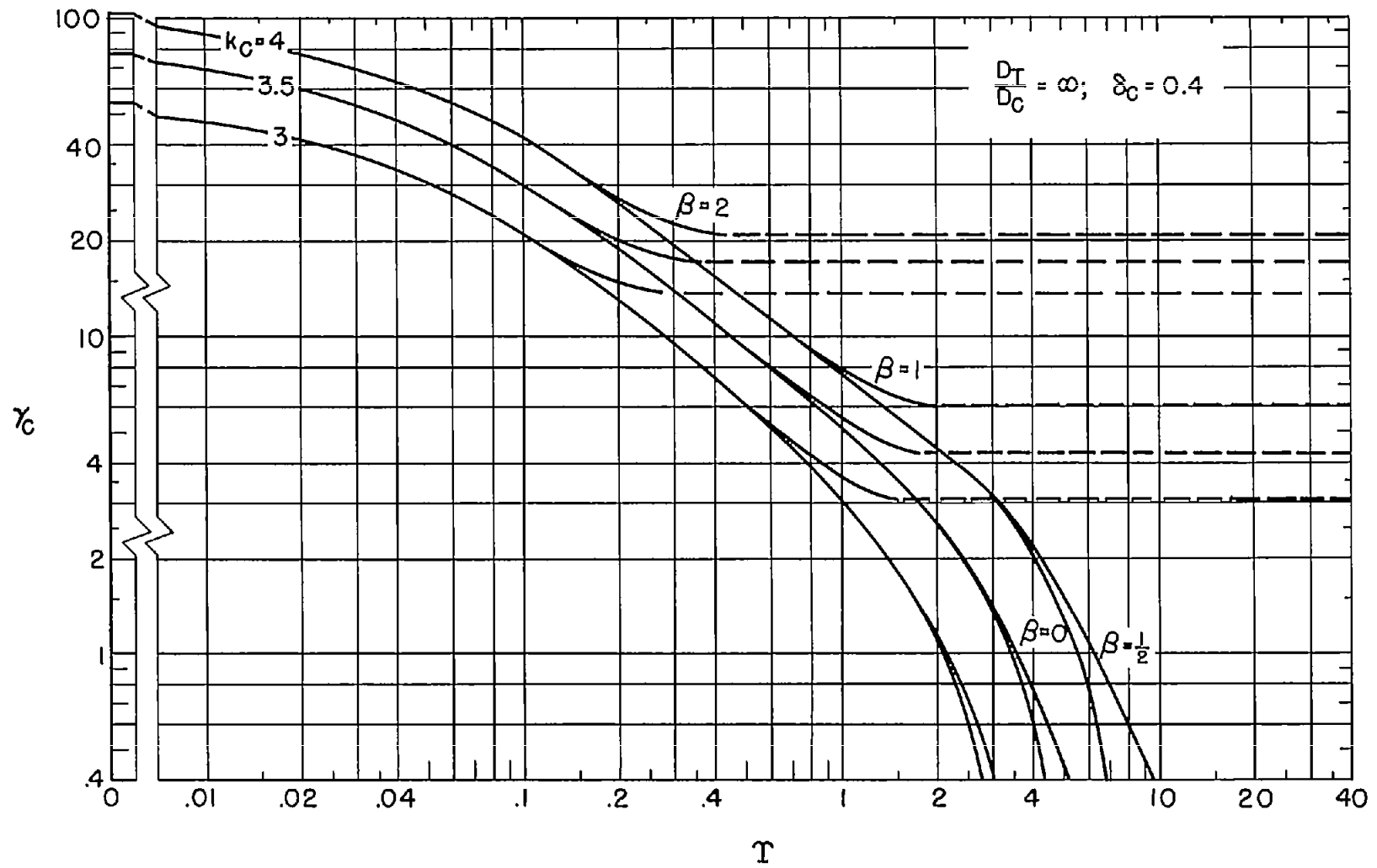


Figure 10.—Interaction of stringer stiffness and post stiffness. $\frac{D_T}{D_C} = \infty$; $\delta_C = 0.4$.

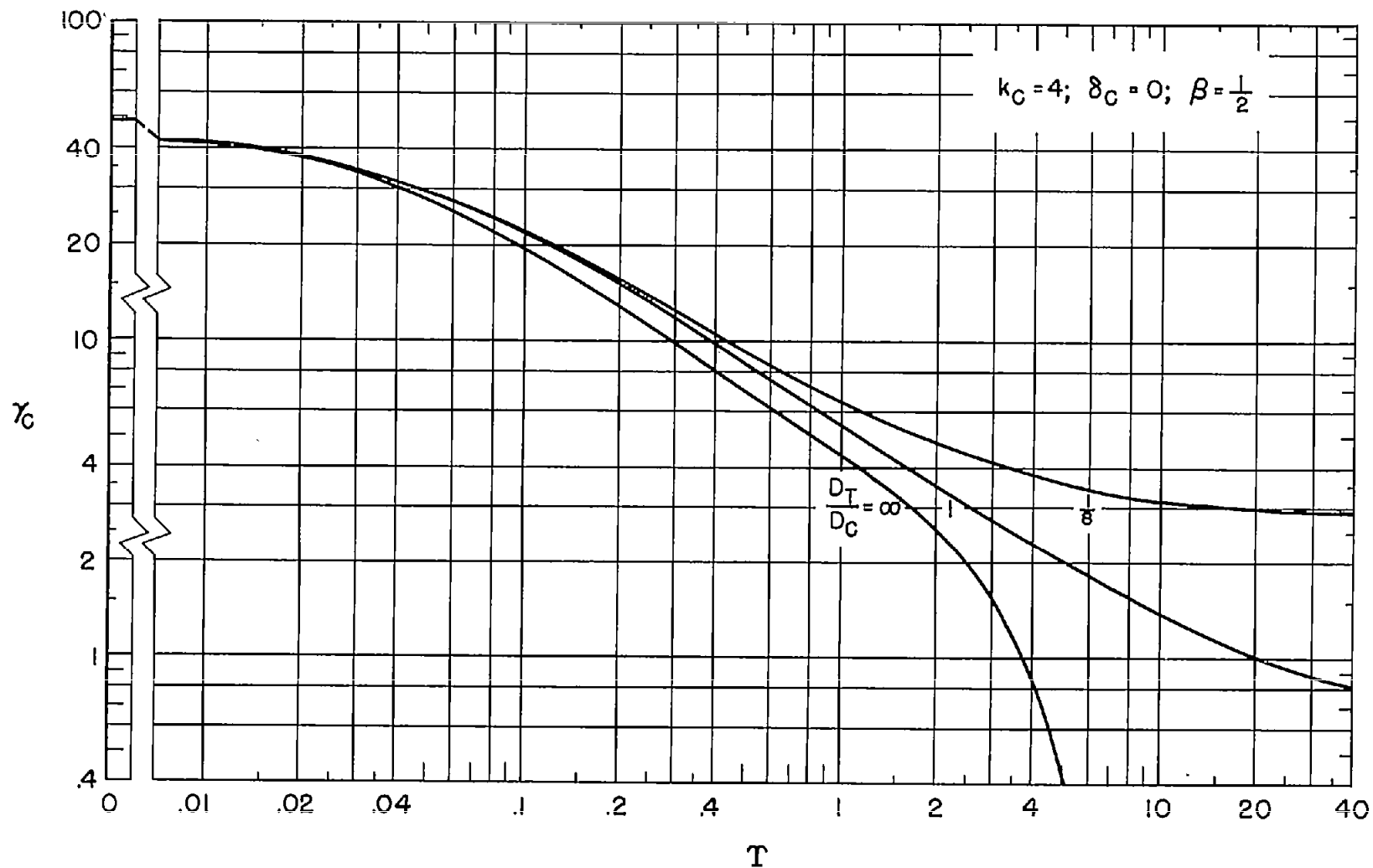


Figure 11.—Effect of tension cover stiffness on interaction of stringer stiffness and post stiffness.

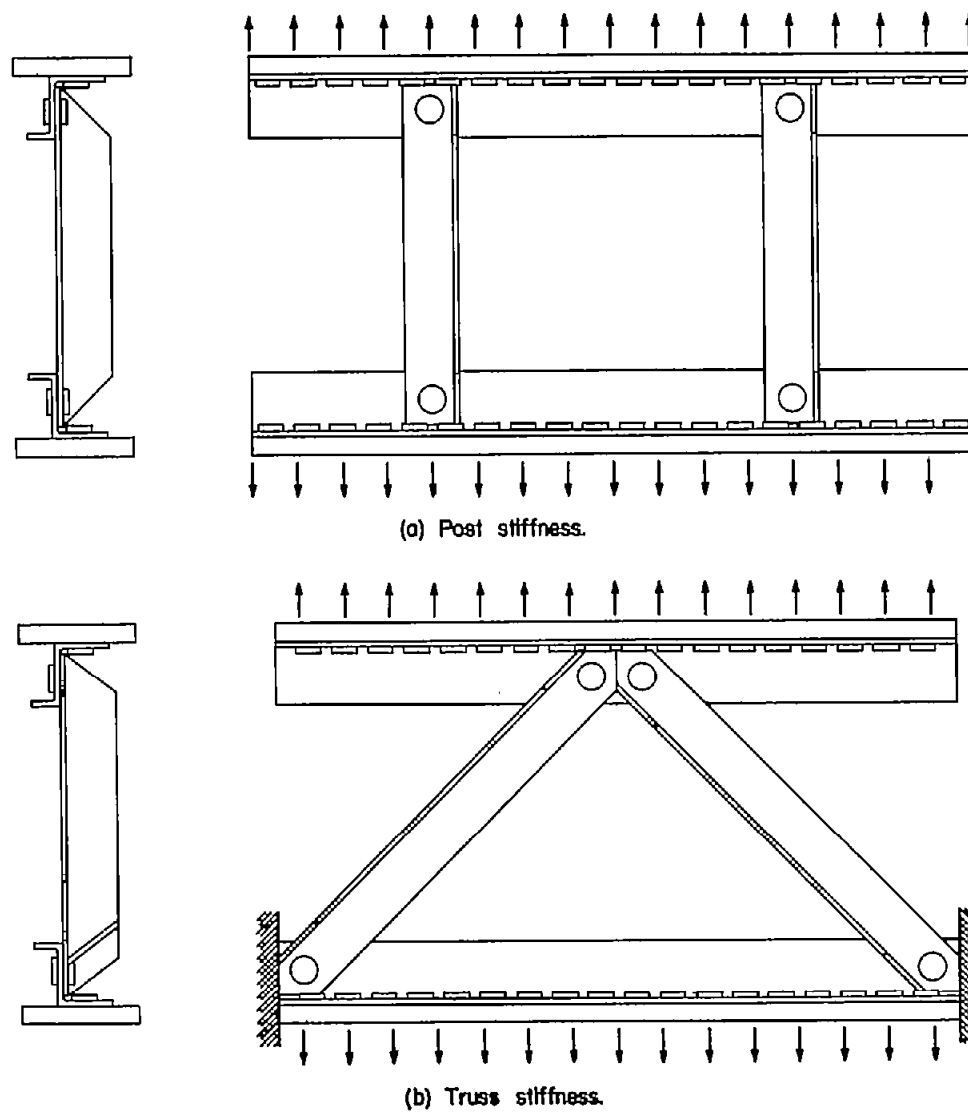


Figure 12.—Test for evaluating effective support stiffness.

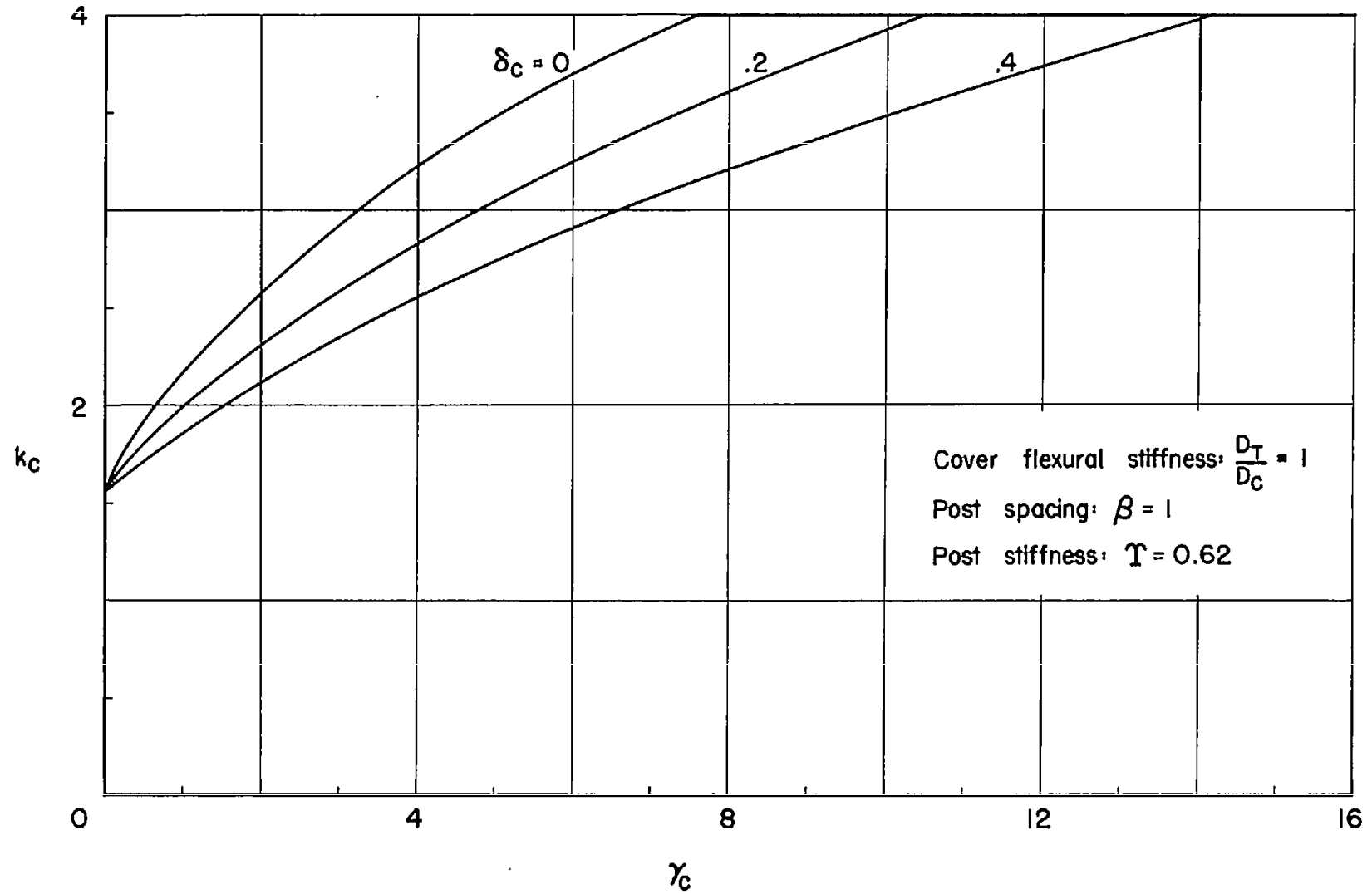


Figure 13.- Effect of stringers on the buckling-stress coefficient for post-stiffened beams.